Abstract
This work presents a fully three-dimensional methodology for the computational analysis of the interaction between catenary and pantographs. The finite element method is used to support the modeling of the catenary while a multibody dynamics methodology is applied to support the pantograph modeling. The contact between the two subsystems is described using a penalty contact formulation. An high-speed co-simulation procedure to ensure the communication between the two methodologies is proposed. The application of the proposed methodology to the study of the interaction of multiple pantographs of a train with a catenary demonstrates the influence of the leading pantograph over the quality of contact of the rear pantograph.

Introduction
The limitation on the top velocity of high-speed trains concerns the ability to supply the proper amount of energy required to run the engines, through the catenary-pantograph interface. Due to the loss of contact not only the energy supply is interrupted but also arcing between the collector bow of the pantograph and the contact wire of the catenary occurs, leading to the deterioration of the functional conditions of the two systems. An alternative would be to increase the contact force between the two systems. But such force increase would lead to a rapid wear of the registration strip of the pantograph and of the contact wire with negative consequences on the durability of the systems. Even in normal operating conditions, a control on the catenary-pantograph contact force is required to ensure longer maintenance cycles and a better reliability of the systems. The foreseeable developments of active pantographs suggest new forms of controlling the energy supply for the high speed trains. But all these situations require that the dynamics of the pantograph-catenary are properly modeled and that software used for analysis, design or to support maintenance decisions is not only accurate and efficient but also allows for modeling all details relevant to the train overhead energy collector operation.

A large number of works dedicated to the study of the catenary-pantograph interaction are being presented to different communities emphasizing not only the mechanical aspects of construction, operation and maintenance but also the challenges for simulation due to the multiphysics characteristic of the problem. Gardou [1] presents a rather simple model for the catenary, using 2D finite elements, where all nonlinear effects are neglected. The single model of catenary analyzed is excited by a lumped mass model of a pantograph. Jensen [2] presents a detailed study on the wave propagation problem on the catenary and a 2D model for the catenary-pantograph dynamics. In a similar line of work Dahlberg [3] describes the contact wire as an axially loaded beam and uses modal analysis to represent its deflection when subjected to transversal and axial loads, showing in the process its relation to the critical velocity of the pantograph. In both references [1] and [2] not only the representation of the contact forces is not discussed but also no reference is made on how the integration algorithms are able to handle the contact loss and impact between registration strip and contact wire. Labergri [4] presents a very thorough description of the pantograph catenary system that includes a 2D model for the catenary based on the finite element method, and a pantograph model based on a multibody approach, being the contact treated by unilateral constraints. In all works mentioned it is claimed that the catenary structural deformations are basically linear and, consequently, the catenaries are modeled using linear finite elements, except for the droppers’ slacking which is handled as a nonlinear effect but not by nonlinear finite elements. Both Seo, et al. [5,6] state the need to treat the catenaries as being nonlinear due to their large deformations. They treat the catenary contact wire with a finite elements based on the absolute nodal coordinate formulation while the pantograph is a full 3D multibody model. The contact is represented by a kinematic constraint
between contact wire and registration strip and no loss of contact is represented. None of the models used has been validated and no comparative studies are provided to support the claims regarding the need to handle nonlinear catenary deformations or the suitability of using linear deformations only.

Most of the works focusing the pantograph-catenary interaction elect the finite element method to develop and analyze linear models catenaries and use lumped mass pantograph models due to the need to maintain the linearity of the analysis. However, it is recognized by a large number of researchers that the nonlinearities of the pantograph system play a very important role in the energy collection and, therefore, either nonlinear finite element or multi-body models can deliver superior analysis capabilities [5-12]. Due to the multiphysics problem involved in modeling the catenary-pantograph system and the need for its simulation Arnold and Veitl [8] suggest the co-simulation between the finite difference discretization of the catenary and the multibody representation of the pantograph. Mei, Zhang et al. [9,10] suggest a coupling procedure between a finite element discretization of the catenary and a physical prototype of a pantograph. This work shows the possibility of coupling numerical and experimental techniques. Rauter et al. [11] show how the coupling between finite element software, to solve the dynamics of the catenary, and multibody software, to obtain the dynamic response of the pantograph, can be efficiently achieved. In these references it is observed that the finite element code ANSYS [13] is the most popular choice of software for the catenary [7,9,10,12] while no major preferences for a particular multibody code are stated.

There are, currently, no accepted general numerical tools designed to simulate the pantograph-catenary system in nominal, operational, and deteriorated conditions. Here it is understood that operating conditions must take into account the wear effects and the deteriorated conditions include extreme climatic conditions, material defects or mechanical problems. Several important efforts have been reported to understand the mechanisms of wear in catenaries and the effect of defect conditions on the dynamics of the complete system by Collina and Bruni [14] and by Collina, et al. [15] or to describe the aerodynamic effects on the quality of the catenary-pantograph contact [16]. The dynamic analysis procedures and the models developed for catenaries and pantographs are also used for designing pantograph control paradigms [17,18] or even wire-actuator control and contact force observers [19].

The different computational procedures and methods developed for representing the pantograph-catenary interaction led to the development of several computer programs used by designers and analysts. The code CATMOS [20], developed in the early 1990s, allows for the vibration analysis of the system. The pantograph is represented by a lumped-mass model and the catenary by Euler-Bernoulli and Timoshenko beam elements. No nonlinearities are considered in the system. Using finite element models in the framework of the nonlinear finite element code ABAQUS the program FAMOS [21] enables the development of linear finite element models for the catenary and nonlinear finite element models for the pantograph. This program enables the analysis of fully three dimensional pantograph models. Veitl and Arnold [22] proposed a co-simulation strategy between the code PROSA, where a catenary is described by the finite difference method and the SIMPACK commercial multibody code used to simulate the pantograph. All models involved in this work are 3D but the catenaries are hard coded, and therefore, the models and programs can hardly be used for different catenary systems. The program DINACAT/WINCAT [23] uses the finite element method to represent both catenary and pantograph. This is a two-dimensional program in which the lumped-mass pantograph models are used.

The methodology presented here, developed in the framework of the EUROPAC project, uses co-simulation between a finite element code to describe catenaries, EUROPACAS-FE, and a multibody codes to handle the dynamics of pantographs, EUROPACAS-MB. The co-simulation strategy is achieved by using memory based communication only and efficiently coordinating the integration of the two dynamic subsystems, catenary and pantograph models. The application to a high-speed train with multiple pantographs running on a suitable catenary is used to demonstrate the methods described here.
2 Multibody dynamics: direct solution

A typical multibody model is defined as a collection of rigid or flexible bodies that have their relative motion constrained by kinematic joints and is acted upon by external forces. The forces applied over the system components may be the result of springs, dampers, actuators or external applied forces describing gravitational, contact/impact or other forces. Let the configuration of the multibody system be described by $n$ Cartesian coordinates $q$, and a set of $m$ algebraic kinematic independent holonomic constraints $\Phi$. A system of differential algebraic equations, representing the constrained equations of motion is defined as \[ \begin{bmatrix} M & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} g \\ \gamma \end{bmatrix} \quad (1) \]

where $M$ is the mass matrix, $\Phi_q$ the Jacobian matrix, $\dot{q}$ the acceleration vector, $\lambda$ the vector of the Lagrange multipliers, $g$ a vector with the forces applied on the system body and the terms dependent on the system velocities and $\gamma$ is a vector with the right-hand side of the constraint acceleration equations. Equation (1) has to be solved for $\dot{q}$ and a unique solution is obtained when the constraint equations are considered simultaneously with the differential equations of motion and a proper set of initial conditions [25,26]. In each integration time step, the accelerations vector, $\dot{q}$, together with velocities vector, $q$, are integrated to obtain the system velocities and positions at the next time step. This procedure is repeated until final time is reached.

The multibody system for the pantograph has to be integrated for long analysis periods requiring efficient and stable numerical procedures. The set of differential algebraic equations of motion (1) does not use explicitly the position and velocity equations associated to the kinematic constraints. Consequently, for moderate or long time simulations, the original constraint equations are rapidly violated due to the integration process. Thus, in order to stabilize or keep under control the constraints violation, equation (1) is solved by using the Baumgarte Stabilization Method [27] or the Augmented Lagrangean formulation [28], and the integration process is performed using a predictor – corrector algorithm with variable step and order [25,26]. Furthermore, due to the long time simulations typically required for pantograph-catenary interaction analysis, it is also necessary to implement constraint violations correction methods. The Coordinate Partition method is used for the purpose [24].

Finite element dynamic analysis

The motion of the catenary is characterized by small rotations and small deformations in which the only nonlinear effect is the slacking of the droppers. Using the finite element method to represent the structure, the equilibrium equations for the structural system are \[ M\ddot{x} + C\dot{x} + Kx = f \quad (2) \]

where $M$, $C$ and $K$ are the finite element global mass, damping and stiffness matrices, respectively, $\dot{x}$ is the vector with the nodal displacements, $\dot{v}$ is the vector of nodal velocities, $\dot{a}$ is the vector of nodal accelerations and $f$ is the vector with the applied forces. Equation (2) needs to be solved for $x$ or for $\dot{a}$ depending on the integration method used to obtain the structural system dynamic response. In this work the integration of the nodal accelerations is achieved using a Newmark family integration algorithm [30].

Contact model for catenary-pantograph

The contact force due to pantograph-catenary interaction, regarding present operating conditions and pantograph and catenary technology, is characterized by a high-frequency oscillating force with high relative amplitude. Railway industry measurement data shows that reasonable values for the contact force are, for a train running at approximately 80 m/s: a mean value of 200N oscillating between 400N and 100N. Loss of contact in particular points of the catenary may also occur. Therefore impact effects must be included in the model.
Although different continuous force contact force models may have different aspects they all have similar features, i.e., they evaluate the contact force as a function of a pseudo-penetration between two elements and a proportionality factor often designated as stiffness of the contact elements. The contact model, exemplified here as one of the typical contact force models that is both accurate and computationally efficient, is based on the work of Hunt and Crossley [31] and it has been proposed by Lankarani and Nikravesh [32]. This contact force model also includes hysteresis damping for impact between components of a system or with external components to the system. In this work, the Hertzian type contact force including internal damping can be written as [32]

\[ F_n = K\delta^2 \left[ 1 + \frac{3(1-e^2)}{4} \frac{\delta}{\delta_{\text{imp}}} \right] \]  

(3)

where \( K \) is the generalized stiffness contact, \( e \) is the restitution coefficient, \( \delta \) is the relative penetration velocity and \( \delta_{\text{imp}} \) is the relative impact velocity. The proportionality factor \( K \) is obtained from the Hertz contact theory as the external contact between two cylinders with perpendicular axis.

In the contact law it is implied that the pseudo-penetration \( \delta \) is known, or calculated based on the position of the elements in contact. Then the force is evaluated based on the material geometric properties, \( K \) and \( e \), and on the kinematic variables, which is applied, in turn, to finite element mesh, in the EUROPACAS-FE code, and to the multibody model, in the EUROPACAS-MB software. Note that the calculation and application of the contact forces in each subsystem has different implications on the integration algorithms that are further explored in this work.

**Co-simulation structure**

The analysis of the pantograph-catenary interaction is done by two independent codes, EUROPACAS-MB, which uses a multibody formulation, and the EUROPACAS-FE that is a finite element software. Both programs can work as stand-alone codes. The structure of the communication between the codes is shown in Figure 1. The EUROPACAS-MB code provides the EUROPACAS-FE code with the positions and velocities of the pantographs registration strips. EUROPACAS-FE calculates the contact force, using the contact model represented by equation (3), and the location of the application points in the pantographs and catenary, using geometric interference. These forces are applied to the catenary, in the finite element code, and to the pantograph model, in the MB code. Each code handles separately the equations of motion of each sub-system based on the shared force information.

![Figure 1: Structure of the communication scheme between the MB and the FE codes](image)

The typical numerical integration algorithms used by FEM codes are Newmark family algorithms [11]. Therefore the FEM code needs a prediction of the positions and velocities not only of the catenary but also of the pantograph in a forthcoming time before advancing to a new time step. A predicted contact force is calculated and, using the finite element method equations of equilibrium, the catenary accelerations are computed for the new time-step. The calculated acceleration values are used to correct the initially predicted positions and velocities of the catenary. The MB code uses a Gear multi-step multi-order integration algorithm [25,26]. To proceed with the dynamic analysis, the MB code needs information about the positions and velocities of the pantograph components and also the contact force and its application point coordinates at different time instants during the integration time period, and not only at its start and end. The
compatibility between the two integration algorithms imposes that the state variables of the two subsystems are readily available during the integration time but also that a reliable prediction of the contact forces is also available at any given time step. Several strategies can be envisaged to tackle this co-simulation problem such as the gluing algorithms proposed by Hulbert, et al. [33] or the co-simulation procedures suggested by Kubler and Schiehlen [34].

The key of the synchronization procedure between the MB and FE codes is the time integration, which must be such that it is ensured the correct dynamic analysis of the pantograph-catenary system, including the loss and regain of contact. Let it be assumed that the FE integration code is of the Newmark family and has a constant time step. Moreover, let it be assumed that the time step of the FE is small enough not only to assure the stability of the integration of the catenary but also to be able to capture all starts of contact between the pantograph registration strip and the contact wire of the catenary. The only restriction that is made to the integration algorithm of the multibody code is that its time step cannot exceed the time step of the FE code. Finally let it be assumed that both codes can start independently from each other, i.e., the catenary FE model and the pantograph MB model include the initial conditions for the start of the analysis expressed in terms of the initial positions and velocities of all components of the systems. Nowhere in the communication procedure outlined it is implied what kind of integration algorithm is used for the FE catenary analysis, provided that it is a fixed time step integrator. Even this condition can be relaxed, but it would not have any practical implication as it is not usual that FE dynamic analysis is performed with variable time step algorithms.

**Application to the study of high-speed trains with multiple pantographs**

The methodology is applied to the simulation of a high-speed train equipped with two pantographs running on a catenary, according to the scenario represented in Figure 2, with a speed of 300 km/h. The CX pantograph equips the train while the SNCF High Speed 25kV catenary is used to provide the power to the train.

In all simulations considered here the train is equipped with two CX pantographs, represented in Figure 3. The multibody model of this pantograph is described in reference [35]. The catenary used in this simulation scenario includes the contact wire, which hangs from the droppers and the steady arms, a stitch wire, which in turn is hanging from the messenger wires supported in the bracket, as shown in Figure 3. A bracket and a mast are hinged to each support, spaced from each other by 54 m, being the registration arm connected to the mast, and the steady arm hinged to the registration arm. The catenary geometry is fully three dimensional being the stagger imposed by the location of the steady arms.

![Figure 2. Train equipped with two pantographs.](image-url)  

![Figure 3. CX pantograph and SNCF High Speed 25kV](image-url)
The contact forces of the pantographs on the catenary are presented in Figure 4 considering their separation by different amounts. The observation of the contact force, corresponding to the length run by the pantograph between two consecutive registration arms of the catenary, indicates that the front pantograph contact force is similar to that of the case of the train equipped with a single pantograph. The rear pantograph contact force presents contact forces with much larger oscillations than the front pantograph. Not only has the rear pantograph higher contact forces but it also has the lowest.

Figure 4. Contact force results (filtered at 20Hz) of the CX pantographs, for the scenario of a train equipped with two pantographs separated by: (a) 40 m; (b) 100 m; (c) 160 m; (d) 200 m

It is observed that for a separation between pantographs of 40 m contact losses occur between the catenary supports for the rear pantograph. However, as the separation between pantographs increases the contact conditions of the rear one become more similar to those of the front pantograph. The exception seems to be for a separation of 200 m, for which the number of contact loss between the rear pantograph and catenary increase. That seems to be due to the frequency of the passing pantographs matches a natural frequency of the catenary. The results for the catenary-pantographs interactions show that the leading pantograph contact force is, in average, 143 N while for the rear pantograph an average contact force is 166 N. However, the quality of contact of the rear pantograph is much lower, as shown in Figure 4.

Figure 5 presents the Z coordinate of the pantograph contact point with the catenary, for a pantograph separation of 200 m. These results confirm the larger amplitude of the vertical travel of the rear pantograph. Figure 6 shows the displacement of a dropper and a registration arm of the catenary due to the pantographs. It is observed an average amplitude of 25 cm for the dropper and of 12 cm for the steady arm vertical displacements. It is also observed in figure 6 that the structure of the catenary is very lightly damped.
Conclusions

Fully three dimensional methodology for the analysis and modeling of catenaries and pantographs, each using the most suitable methodology, have been presented in this work. The finite element method is used for the catenary while a multibody methodology is used for the pantograph in a completely independent program being the communication between the two programs the crucial element. This communication has been developed such a way that any choice for integrators, for both FE and MB codes, can be made. The methodology has been demonstrated through the application to a high-speed train equipped with two pantographs. The application clearly shows that the operation of the front pantograph has important interferences on the quality of the rear pantograph contact, eventually leading to frequent contact losses, especially is the spacing between the pantographs is small or if the frequency at which the pantographs pass by the catenary supports is similar to a natural frequency of the catenary.

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